MODELING OF SOIL BEHAVIOR BASED ON $t_i$ CONCEPT

Teruo Nakai
Department of Civil Engineering, Nagoya Institute of Technology, Nagoya, Japan, 466-8555
nakai.teruo@nitech.ac.jp

ABSTRACT: Simple methods to describe some typical soil properties, which cannot be taken into consideration properly by Cam-clay model, are presented. The influence of intermediate principal stress is considered using the concept of modified stress $t_i$. For considering the influence of density and/or confining pressure, subloading surface concept is introduced, and a new methodology using this concept is elaborated. The influence of stress path on the direction of plastic strain increments is considered by dividing the plastic strain increment into two components using only one yield surface. Extending the concept of what was employed in the consideration of the influence of density, a method to consider the influence of the bonding effect on soil behavior is presented. It is shown that the behavior of structured soils such as natural deposited clay can be described by considering the influence of density and the influence of bonding. In the present model, only two material parameters $a$ and $b$ for representing the influence of density and the influence of bonding, respectively, are added to the parameters of the previous model, which are fundamentally the same as those of the Cam-clay model. The validity of the present model is confirmed by the results of the monotonic and cyclic loading tests not only on normally and over consolidated clay but also on loose and dense sands in three-dimensional stresses and numerical simulations of the structured soils.

1. INTRODUCTION

Most of practical designs of earth structures, foundations, countermeasure against earth disaster and others have been made based on elastic theory and/or rigid plastic theory, in which the deformation characteristics of geomaterials such as soil dilatancy are not considered. From the development of the Cam-clay model, non-linear elastoplastic analyses have been carried out to solve boundary value problems. However, applications to practical design have been limited, because most of the constitutive models used in analysis cannot describe the typical soil behaviors comprehensively.

The Cam-clay model, which was developed in Cambridge University (e.g., Schofield and Wroth, 1968), is certainly the first elastoplastic model applicable to the practical deformation analysis of ground, because the model at least describes both behaviors of soils under shear loading and under consolidation. This model is certainly very simple, i.e., the number of material parameters is few, and the meaning of each parameter is clear. However, the following features of soil behavior can not be properly simulated using the conventional Cam-clay model:

(i) Influence of intermediate principal stress on the deformation and strength of soil
(ii) Stress path dependency on the direction of plastic strain increments
(iii) Positive dilatancy during strain hardening
(iv) Behavior of soil under cyclic loading
(v) Influence of density and/or confining pressure on the deformation and strength
(vi) Behavior of structured soil
(vii) Soil anisotropy and non-coaxiality
(viii) Time effect and age effect
(ix) Unsaturated soils

To overcome these problems, many models have been proposed since the Cam-clay model. However, most of them are much more complex than the Cam-clay model, and their applicability to practical problems is still limited.

At the 16th International Conference on Soil Mechanics and Geotechnical Engineering, which was held in Osaka in 2005, there was a Practitioner/Academic Forum, coordinated by Prof. Poulos, "Which is better for practical geotechnical engineering, simplified model or complex model". The opinion of the author of this paper is that a model should be simple and sophisticated (not complicated) so as to be applicable and useful to the practice. This keynote lecture is mostly concerned with this topic.

Simple methods to consider the features (i) to (vi) in particular are presented in the present paper. Here, features (i) and (ii) are taken into consideration without adding material parameter to those of Cam-clay. For considering features (iii) to (v), only one material parameter is added. The behavior of structured soil, feature (vi), is also taken into consideration by one extra material parameter. The validities of the elastoplastic model in which the above features are properly taken into account are confirmed by the results of various kinds of elements tests on sand and clay in the general stress systems and the result of parametric numerical simulation of element tests.
2. OUTLINE OF ORDINARY ELASTOPLASTIC MODELS FOR SOILS

2.1 Cam-clay model

Most of the constitutive models for soils such as the Cam-clay model (e.g., Schofield and Wroth, 1968; Roscoe and Burland, 1968) have been formulated using stress invariants (mean stress $p$ and deviatoric stress $q$) and the strain increment invariants (volumetric strain increment $d\varepsilon_v$ and deviatoric strain increment $d\varepsilon_d$). These parameters are defined by the normal and parallel components of the stress and the strain increment to the octahedral plane and defined by Eqs. (1) to (4) (see Figs. 1 and 2).

\[
p = \frac{1}{\sqrt{3}} \text{OP} = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3} \sigma_0 \delta_0
\]

\[
q = \frac{1}{\sqrt{2}} \text{NP} = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]
\]

\[
d\varepsilon_v = \sqrt{3} \text{ON} = d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = d\varepsilon_0 \delta_0
\]

\[
d\varepsilon_d = \frac{2}{\sqrt{3}} \text{NP} = \frac{2}{\sqrt{3}} \left( (d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_3 - d\varepsilon_1)^2 \right)
\]

Then, isotropic hardening elastoplastic models for soils are usually formulated by using the yield function and/or plastic potential, which is given by a function of mean stress $p$ and stress ratio $\eta$ ($=q/p$) in the following form:

\[
f = f(p, \eta = q/p, \rho_1) = \ln p + \varsigma(\eta) - \ln \rho_1
\]

where, $\varsigma(\eta)$ is an increasing function of $\eta$ and satisfies the condition $\varsigma(0) = 0$, $\rho_0$ is the value of the initial yield surface at $p$-axis, and $\rho_1$ determines the size of the current yield surface (the value of $p$ at $\eta = 0$). Then, the plastic strain increment can be calculated by assuming the associated flow rule in $\sigma_{ij}$ space

\[
d\varepsilon_p^e = \Lambda \frac{\partial f}{\partial \sigma_\eta} = \Lambda \left( \frac{\partial f}{\partial p} \frac{\partial p}{\partial \sigma_\eta} + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial \sigma_\eta} \right)
\]

From Eqs. (5) and (6), the following relation between stress ratio and plastic strain increment ratio (stress-dilatancy relation) is obtained:

\[
\frac{d\varepsilon_p^e}{d\varepsilon_d^e} = \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial q} = 1 - \frac{\varsigma(\eta)}{\varsigma(\eta)}
\]

Throughout this paper, superscripts $p$ and $e$ imply the plastic and elastic components of the quantities. The stress ratio function $\varsigma(\eta)$ in Eq. (5) is given as follows for the original Cam-clay model (Schofield and Wroth, 1968) and the modified Cam-clay model (Roscoe and Burland, 1968):

\[
\varsigma(\eta) = \frac{1}{M} \eta \quad \text{(original)}
\]

\[
\varsigma(\eta) = \ln \frac{M^2 + \eta^2}{M^2} \quad \text{(modified)}
\]

Here, $M$ is the stress ratio, $\eta$ at critical state. The stress-dilatancy relation for two models are then given by

\[
\frac{d\varepsilon_p^e}{d\varepsilon_d^e} = M - \eta \quad \text{(original)}
\]
\[ \frac{d\varepsilon_p^p}{d\varepsilon^{pp}} = \frac{M^2 - \eta^2}{2\eta} \] (modified) \hfill (11)

The shape of the yield surfaces in \( p-q \) plane and the direction of plastic strain increments are shown in Fig. 3, and the stress-dilatancy relation for the two models are shown in Fig. 4. Also, the shape of yield surface which is formulated using \( p \) and \( q \) are inevitably circular on the octahedral plane so that the direction of plastic strain increment on the octahedral plane is always in the radial direction as shown in Fig. 5.

The strain hardening parameter \( p_1 \), which represents the size of the yield surface, is related to the plastic volumetric strain \( \varepsilon_v^p \) as
\[ \varepsilon_v^p = C_p \ln \frac{p}{p_0} \quad \left( C_p = \frac{\lambda - \kappa}{1 + \epsilon_0} \right) \] \hfill (12)
in which \( \epsilon, \lambda, \) and \( \kappa \) denote void ratio, compression index and swelling index, respectively, and \( \epsilon_0 \) is the void ratio at reference state \( (p=p_0 \text{ and } \eta=0) \).

Therefore, the yield function of Eq. (5) can be rewritten as follows using the function of stress invariants \( F(p, \eta=q/p) \) and the function of strain hardening parameter \( H(\varepsilon_v^p) \):
\[ f = F(p, \eta) - H(\varepsilon_v^p) = 0 \] \hfill (13)

Here, \( F(p, \eta) \) and \( H(\varepsilon_v^p) \) are given by
\[ F(p, \eta) = \ln \frac{p}{p_0} + \zeta(\eta) \] \hfill (14)
\[ H(\varepsilon_v^p) = \frac{1}{C_p} \varepsilon_v^p \] \hfill (15)

From the compatibility condition \( (df=0) \), the proportionality constant \( \Lambda \) is given by
\[ \Lambda = \frac{\left( \frac{\partial F}{\partial \sigma_{ii}} \right) d\sigma_{ii}}{\left( \frac{\partial F}{\partial \sigma_{ii}} \right) \left( \frac{\partial F}{\partial \delta_{ii}} \right)} = \frac{dF}{\partial \sigma_{ii}} \frac{dF}{\partial \delta_{ii}} \] \hfill (16)

Here, \( h_p \) represents the plastic modulus.

The elastic strain increment is given by the generalized Hooke’s law
\[ d\varepsilon_e^p = \frac{1+\nu}{E_e} d\sigma_{ii} - \frac{\nu}{E_e} d\sigma_{ii} \delta_{ii} \] \hfill (17)

Young’s modulus \( E_e \) is expressed in terms of the swelling index \( \kappa \) and Poisson’s ratio \( \nu \) as
Therefore, the total strain increment is given by
\[ d\varepsilon_{ij} = d\varepsilon_{ij}^p + d\varepsilon_{ij}^e \] (19)
The loading condition is expressed as follows:
\[
\begin{cases}
  d\varepsilon_{ij}^p \neq 0 & \text{if } f = 0 \& \Lambda = \frac{df}{k'} > 0 \\
  d\varepsilon_{ij}^p = 0 & \text{otherwise}
\end{cases}
\] (20)
and the yield surface is fixed when no plastic strain occurs.

### 2.2 Consideration of three-dimensional stress-strain behavior of clay and sand

Figures 6 and 7 show the observed results of drained triaxial compression \((\sigma_1 > \sigma_2 = \sigma_3)\) and triaxial extension \((\sigma_1 = \sigma_2 > \sigma_3)\) tests on normally consolidated Fujinomori clay and medium dense Toyoura sand under constant mean principal stress, in terms of the relation between stress ratio \((q/p)\), deviatoric strain \((\varepsilon_d)\) and volumetric strain \((\varepsilon_v)\). It can be seen in these figures that the deformation and strength of soils in three-dimensional (3D) stress conditions can not be described uniquely using these invariants. Figures 8 and 9 show the observed stress-dilatancy relation of the same tests, which is arranged using the above stress and strain increment invariants.

Although the strain increment ratio in these figures is arranged including the elastic components, the plastic strain increment ratio can be considered as almost the same as the ratio of the total strain increments, because the elastic strain increments are much smaller than the plastic strain increments under shear loadings. There is no unique relation between \(d\varepsilon_v/d\varepsilon_d\) and \(q/p\) in Figs. 8 and 9 as the shape of yield surface on \(p-q\) plane is dependent on the relative magnitude of the intermediate principal stress. Figures 10 and 11 show the directions and the magnitudes of the observed shear strain increments on the octahedral plane in the true triaxial \((\sigma_1 > \sigma_2 > \sigma_3)\) tests \((\theta=15^\circ, 30^\circ\) and \(45^\circ)\). Here, the length of each line is proportional to the value of shear strain increment divided by the shear-normal stress ratio increment on the octahedral plane. In the figures, \(\theta\) denotes the angle between \(\sigma_1\)-axis and the corresponding radial stress path on the octahedral plane, where \(\theta=0^\circ\) and \(60^\circ\) represent the stress path under triaxial compression and triaxial extension conditions, respectively. Note that the direction of the observed shear strain increments deviates leftward from the direction of shear stress (radial direction) with the increase of stress ratio under three different principal stresses. Since the shape of plastic potential (yield surface) is circle on the octahedral plane as shown in Fig.5 and is formulated using the stress invariants \(p\) and \(q\), such deviation of strain increments in this direction cannot be described.

\[ E_\varepsilon = \frac{3(1-2\nu_j)(1+\varepsilon_0)p}{\kappa} \] (18)
3. INTRODUCING THE INFLUENCE OF INTERMEDIATE PRINCIPAL STRESS IN MODELING

3.1 Definition of modified stress tensor $t_{ij}$ and its stress and strain increment invariants

To describe the deformation and strength characteristics of soils uniquely in 3D stress conditions, Nakai and Matsuoka (1983) proposed an extended concept of the spatially mobilized plane (SMP*). Based on the generalized concept of the SMP*, Nakai and Mihara (1984) developed a method to formulate the elastoplastic model in which the influence of intermediate principal stress can be automatically taken into consideration, by introducing the modified stress $t_{ij}$. In $t_{ij}$ concept, attention is paid to the so-called spatially mobilized plane (SMP; Matsuoka and Nakai, 1974) instead of the octahedral plane in the ordinary concept. The plane ABC in Fig. 12 is the spatially mobilized plane in three-dimensional space, where axes I, II and III imply the direction of three principal stresses. At each of three sides AB, AC and BC of plane ABC, the shear-normal stress ratio is maximized between two principal stresses as shown in Fig. 13. It can be seen that the values of coordinate axes intersected by plane ABC (SMP) are proportional to the square of the ratio between the corresponding principal stresses, because the following equation holds:

$$\tan \left( \frac{45^\circ + \phi_{n_{ij}}}{2} \right) = \frac{1 + \sin \phi_{n_{ij}}}{\sqrt{1 - \sin \phi_{n_{ij}}}} \frac{\sigma_i}{\sigma_j} \quad (i, j = 1, 2, 3; i < j)$$

Therefore, the SMP coincides with the octahedral plane only under isotropic stress condition and varies with possible change of stress ratio. The direction cosines ($a_1, a_2$ and $a_3$) of the normal to the SMP, and the unit tensor
whose principal values are determined by these direction cosines are given by

\[ a_i = \sqrt{\frac{I_i}{I_2} \sigma_i}, \quad a_j = \sqrt{\frac{I_j}{I_2} \sigma_j}, \quad a_k = \sqrt{\frac{I_k}{I_3} \sigma_k} \]  
\[ (22) \]

\[ \sigma_j = \frac{I_1}{I_2} \cdot r_j^{-1} = \frac{I_1}{I_2} (\sigma_a + I_3 \delta_a) (I_2 \sigma_{a_j} + I_3 \delta_{a_j})^{-1} \]  
\[ (23) \]

where \( \sigma_i (i=1,2,3) \) are the three principal stresses, \( I_1, I_2, \) and \( I_3 \) are the first, second and third invariants of \( \sigma_i \), and \( I_{r1}, I_{r2} \) and \( I_{r3} \) are the first, second and third invariants of \( r_{ij} \), which is the square root of the stress tensor or \( r_{d} = \delta_{a} \). These invariants are expressed using principal stresses and stress tensors as

\[ I_1 = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_n \]
\[ I_2 = \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1 = \frac{1}{2} \left( \sigma_n^2 - \sigma_n \phi \right) \]  
\[ (24) \]

\[ I_3 = \sigma_1 \sigma_2 \sigma_3 = e_{ij} \sigma_j \sigma_i \sigma_k \]

\[ I_{r1} = \sqrt{\sigma_1 + \sigma_2 + \sigma_3} = r_g \]
\[ I_{r2} = \sqrt{\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1} = \frac{1}{2} \left( r_n^2 - r_n \phi \right) \]
\[ I_{r3} = \sigma_1 \sigma_2 \sigma_3 = e_{ij} r_i r_j r_k \]  
\[ (25) \]

As can be seen from the above equation, \( a_j \) is a function of stress ratio and its principal axes coincide with those of \( \sigma_i \). The modified stress tensor \( t_j \) is then defined by the product of \( a_{jk} \) and \( \sigma_j \) as follows:

\[ t_j = a_{jk} \sigma_j \]  
\[ (26) \]

Its principal values are given by

\[ t_1 = a_1 \sigma_1, \quad t_2 = a_2 \sigma_2, \quad t_3 = a_3 \sigma_3 \]  
\[ (27) \]

In the ordinary models, the stress invariants (\( p \) and \( q \)) and strain increment invariants (\( d\varepsilon \) and \( d\gamma \)) are given by the normal and parallel components of the ordinary stress and strain increment with respect to the octahedral plane (see Figs. 1 and 2). On the other hand, the stress parameters (\( t_1 \) and \( t_2 \)) and strain increment invariants (\( d\varepsilon^\ast \), \( d\gamma^\ast \)) in the \( t_j \) concept are defined as the normal and parallel components of the modified stress \( t_j \) and the strain increment to the SMP (see Figs. 14 and 15). These parameters are, hence, given by

\[ t_j = ON = t_j a_1 + t_j a_2 + t_j a_3 = t_j a_{ij} \]  
\[ (28) \]

\[ t_3 = NT = \sqrt{t_1^2 + t_2^2 + t_3^2 - (t_1 a_1 + t_2 a_2 + t_3 a_3)^2} = \sqrt{t_3^2 - (t_j a_{ij})^2} \]  
\[ (29) \]

\[ d\varepsilon^\ast = ON = d\varepsilon a_1 + d\varepsilon a_2 + d\varepsilon a_3 = d\varepsilon a_{ij} \]  
\[ (30) \]

\[ d\varepsilon^\ast = NT = \sqrt{d\varepsilon^1 + d\varepsilon^2 + d\varepsilon^3 - (d\varepsilon a_1 + d\varepsilon a_2 + d\varepsilon a_3)^2} = \sqrt{d\varepsilon^2 - (d\varepsilon a_{ij})} \]  
\[ (31) \]

The dots in Figs. 16 and 17 show the observed stress-dilatancy relation of the same tests in Figs. 8 and 9, arranged in terms of the relation between stress ratio \( t_j/t_3 \) and stress increment ratio \( d\varepsilon^\ast/d\gamma^\ast \) using the above stress and strain increment invariants based on the \( t_j \) concept. It can be seen that though the relation between \( q/p \) and \( d\varepsilon/d\gamma \) in Figs. 8 and 9 is very much influenced by the intermediate principal stress, the stress-dilatancy relation in Figs. 16 and 17 is independent of the intermediate principal stress. A comparison between the stress and strain increment tensors and their invariants used in the ordinary concept and \( t_j \) concept are shown in Table 1.

Fig. 14 Definition of \( t_1 \) and \( t_3 \)

Fig. 15 Definition of \( d\varepsilon^\ast \) and \( d\gamma^\ast \)
the flow rule in the modified stress $t_{ij}$ space instead of the ordinal $\sigma_{ij}$ space. Therefore, the yield function is given in the same form as Eq. (5).

$$f = f(t_{ij}, X = x_{ij}/H, t_{ij}) = \ln t_{ij} + \zeta(X) - \ln t_{ij0}$$

$$= \ln \left(\frac{t_{ij}}{t_{ij0}}\right) + \zeta(X) - \ln \left(\frac{t_{ij0}}{t_{ij0}}\right) = 0$$

(32)

This yield function is rewritten as

$$f = F(t_{ij}, X) - H(\epsilon_*^p) = 0$$

(33)

Here, $F(t_{ij}, X)$ is expressed by the function of stress invariants based on $t_{ij}$ concept as

$$F(t_{ij}, X) = \ln \left(\frac{t_{ij}}{t_{ij0}}\right) + \zeta(X)$$

(34)

and $H(\epsilon_*^p)$ in Eq. (33) is given by the same function as that of the Cam-clay model.

$$H(\epsilon_*^p) = \frac{1}{C_p} c_*^p$$

(15bis)

Here, $\zeta(X)$ is a monotonically increasing function of $X(x_{ij}/H)$ and satisfies the condition of $\zeta(0)=0$ in the same way as $\zeta(\eta)$ in the ordinary models. Then, plastic strain increment is calculated by the associated flow rule in $t_{ij}$ space as

$$d\epsilon_*^p = \Lambda \frac{\partial F}{\partial t_{ij}} = \Lambda \left(\frac{\partial F}{\partial \sigma_{ij}} \frac{\partial \sigma_{ij}}{\partial \epsilon_*^p} + \frac{\partial F}{\partial X} \frac{\partial X}{\partial \epsilon_*^p}\right)$$

(35)

The proportionality constant $\Lambda$ in the above equation is obtained from the compatibility condition ($df=0$) in the same way as ordinary models.

$$\Lambda = \frac{\left(\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij}\right)}{dF} = \frac{\partial F/\partial \sigma_{ij}}{\partial F/\partial e_*^p} \frac{\partial \sigma_{ij}}{\partial \epsilon_*^p} = \frac{df}{C_p \partial e_*^p}$$

(36)

The derivative of yield function $f$, mean stress $t_{ij}$, and stress variables with respect to $t_{ij}$ and $\sigma_{ij}$ are shown in APPENDIX.

Now, assume that the stress condition moves to the initial state I ($t_{ij}=t_{ij0}$, $X=0$) to the current state P ($t_{ij}=t_{ij0}$, $X=X$) with elastoplastic deformation as shown in Fig. 18. The yield surfaces for both cases are indicated by the broken curve and solid curve. From Eq. (33), there holds the linear relation of $F(t_{ij}, X)=H(\epsilon_*^p)$ in Fig. 19, and the void ratio changes along the normally consolidation line (NCL) in Fig. 20 in the same way as the ordinary models. The loading condition in $t_{ij}$ concept is the same as that in the ordinary model (Eq. (20)). The comparison between the
ordinary model (Cam-clay model) and a model named \( t_{ij} \) clay model, in which \( t_{ij} \) concept is introduced to an ordinary model such as the Cam-clay model, has been presented by Nakai and Matsuoka (1986).

In a recent model (Nakai and Hinokio, 2004), the following equation is adopted for the function of stress ratio \( \zeta(X) \) in Eq. (34):

\[
\zeta(X) = \frac{1}{\beta} \left( \frac{X}{M^*} \right)^{\beta} \tag{37}
\]

When \( \beta = 1 \), the shape of yield function is the same as that of the original Cam-clay model. From Eqs. (34), (35) and (37), the following stress-dilatancy relation holds:

\[
d\varepsilon^p_{ij} = \frac{\partial f}{\partial \varepsilon_{ij}} = \frac{\partial F}{\partial \varepsilon_{ij}} - \frac{\partial \bar{F}}{\partial \varepsilon_{ij}} = 1 - \zeta'(X) \cdot X
\]

where \( \zeta'(X) \) is the derivative of \( \zeta(X) \) with respect to \( X \). Then, the following relation holds:

\[
(M^*)^\beta - X^\beta = \frac{\zeta'(X)}{X^{\beta-1}}
\]

(38)

Figure 21 shows the shape of the yield surface in \( t_N - t_S \) plane together with the direction vector of plastic strain increment which is normal to the yield surface, and Fig. 22 shows the stress-dilatancy relation between \( X = t_S/t_N \) and \( Y = d\varepsilon^p_{ij} / d\varepsilon^p_{ij} \). Here, \( M^* \) implies the intercept with the vertical axis in Fig. 22 and is expressed using \( X_{CS} \) and \( Y_{CS} \), which are the stress ratio \( X \) and \( Y \) at critical state (\( d\varepsilon^p_{ij} = 0 \)).

\[
M^* = (X_{CS} - X_{CS} Y_{CS})^{\beta}
\]

(39)

and \( X_{CS} \) and \( Y_{CS} \) are expressed as follows using the principal stress ratio at critical state in triaxial compression \( R_{CS} = \sigma_1 / \sigma_3 \) (Nakai and Mihara, 1984):

\[
X_{CS} = \frac{\sqrt{2}}{3} \left( \frac{1}{R_{CS}} \right)^{\beta}
\]

(40)

\[
Y_{CS} = \frac{1 - \sqrt{R_{CS}}}{\sqrt{2} (\sqrt{R_{CS}} + 0.5)}
\]

(41)

Figure 23 shows the view of the yield surface in principal space of \( \sigma_1 \) and \( t_{ij} \) (Pedroso et al., 2005). The shape of the yield surface on the octahedral plane in \( \sigma_1 \) space is a rounded triangle and corresponds to that of the SMP.
criterion (Matsuoka and Nakai, 1974) (see diagram (c)). The shape from the hydrostatic axis in \( t_{ij} \) space is also a triangle, though a little more rounded but is not a circle (see diagram (d)). Therefore, the \( t_{ij} \) concept can describe not only the observed uniqueness of stress-dilatancy relation (Figs. 16 and 17) but also the observed deviation of plastic flow from the direction of shear stress on the octahedral plane (Figs. 10 and 11). Employing \( t_{ij} \) concept, in which the yield function is formulated in such a form as Eq. (32) or (33) and the flow rule is assumed in \( t_{ij} \) space as Eq. (35), the influence of intermediate principal stress in constitutive modeling can be automatically taken into consideration.

Figure 24 shows the yield surface of the Cam clay model in \( p-q \) plane in triaxial compression (upper half) and triaxial extension (lower half) conditions. Figure 25 shows the yield surface of the present model in (a) \( t_N-t_S \) plane and (b) \( p-q \) plane. It can be observed that the yield surface of the model based on \( t_{ij} \) concept is symmetric with respect to \( t_{ij} \)-axis but not symmetric with respect to \( p \)-axis. It is also noted that the direction of plastic flow of \( t_{ij} \) model is not normal to the yield surface represented in \( p-q \) plane, because flow rule is assumed in modified stress \( t_{ij} \) space as described above. Also, though the yield surface of the original Cam clay model is not smooth at the tip on \( p \)-axis, the surface of the proposed model is smooth over the whole domain. Such smoothness of the present yield surface is one of the advantages in numerical computations in the same way as the modified Cam clay model (Roscoe and Burland, 1968). Now, the lines in which the minor principal stress \( \sigma_3 \) is zero are indicated in every figure. It can be seen that though models formulated using \( p \) and \( q \) such as the Cam clay model have tension zones (shaded area in Fig. 24) on and inside the yield surface, there is no tension zone in the yield surface formulated using \( t_N \) and \( t_S \). This is because \( \sigma_3=0 \) condition is satisfied on the vertical axis (\( t_S \) axis) in Fig. 25(a). Hence, models based on the \( t_{ij} \) concept not only are capable of describing properly the influence of the intermediate principal stress but also have the above-mentioned benefit for numerical computations.
The effectiveness of this \( t_{ij} \) concept is confirmed by various kinds of element tests not only on normally consolidated clay but also on over consolidated clay and loose and dense sand in the section (4.2).

### 3.3 Meaning of \( t_{ij} \) concept

In this section the meaning of \( t_{ij} \) concept is discussed, focusing mostly on the microscopic point of view. Several researchers have shown that induced anisotropy of soils developed with the change of stresses is characterized by the frequency distribution of the inter-particle contact angles. It has then been shown from microscopic observation (e.g., Oda, 1972) and DEM simulation (e.g., Maeda et al, 2006) that, as the stress ratio increases, the average directions normal to the inter-particle contacts gradually concentrate in the same direction as the major principal stress (\( \sigma_1 \)). Referring to the results of biaxial tests on a stack of photoelastic bars, Satake (1984) pointed out that the principal values (\( \phi_1, \phi_2 \)) of so-called fabric tensor \( \varphi_\sigma \), which represents the relative distribution of the number of the vectors normal to the inter-particle contacts, is approximately proportional to the square roots of the number of the vectors normal to the inter-particle contacts, in 2D condition. Considering an isotropic ground and others. As shown in Eq. (26), the principal values of \( \sigma_{ij} \) are inversely proportional to the square root of the respective principal stresses, therefore:

\[
\frac{\sigma_1}{\sigma_2} \approx \left( \frac{\phi_1}{\phi_2} \right)^{0.5} \tag{42}
\]

Employing a fabric tensor, Satake (1982) also gives the following modified stress tensor \( \sigma_{ij}^* \) for analyzing the behavior of granular materials:

\[
\sigma_{ij}^* = \frac{1}{3} \varphi_\sigma^{-1} \sigma_{ij} \tag{43}
\]

Figure 26(a) shows schematically the distribution of inter-particle contacts in 2D condition. Considering an equivalent continuum, such material exhibits anisotropy since the stiffness in \( \sigma_1 \) direction should be larger than that in \( \sigma_2 \) direction with the increase of stress ratio as shown in diagram (b). When adopting elastoplastic theory, it is reasonable to treat the soil as an isotropic material by introducing the modified stress \( t_{ij} \) in which induced anisotropy is already taken into consideration. This is because the normality rule, for which the direction of plastic flow is normal to the yield surface (plastic potential)) should hold in an isotropic space, like the transformed space used to analyze seepage problem in anisotropic ground and others. As shown in Eq. (22), the principal values of \( a_{ij} \) are inversely proportional to the square root of the respective principal stresses, therefore:

\[
a_1:a_2 = \frac{1}{\sqrt{\sigma_1}} : \frac{1}{\sqrt{\sigma_2}} \tag{44}
\]

It can be noted that \( a_{ij} \) corresponds to the inverse of the fabric tensor in Eq. (43), and \( t_{ij} \) defined by Eq. (26) corresponds to the modified stress induced by Satake (1982). As shown in diagram (c), the stress ratio \( t_1/t_2 \) in the modified stress space is smaller than stress ratio \( \sigma_1/\sigma_2 \) in the ordinary stress space, because \( \sigma_1 \) is smaller than \( \sigma_2 \) as Eq. (44) shows. Then, it is reasonable to assume the flow rule (normality condition) not in \( \sigma_{ij} \) space but in \( t_{ij} \) space, because the condition of the anisotropic material under anisotropic stress ratio in diagram (b) can be considered to be the same as that of the isotropic material under lower stress ratio in diagram (c).

![Fig. 26 Anisotropy and its expression](image)
strain increments referred to the SMP. As a consequence, the influence of the intermediate principal stress can be explained as the characteristics due to the induced anisotropy and frictional law – both are considered using $t_y$ concept.

4  INTRODUCING THE INFLUENCE OF DENSITY AND/OR CONFINING PRESSURE IN MODELING

4.1 Model considering influence of density and/or confining pressure (subloading $t_y$ model)

It is known that the Cam-clay model can describe the strain hardening behavior of soil with negative dilatancy and the strain softening behavior with positive dilatancy under shear loading. However, it is incapable of describing the strain hardening behavior with positive dilatancy of soils such as heavily over consolidated clay and dense sand. To overcome this problem, Asaoka et al. (1997) extended the Cam-clay model to one applicable to over consolidated clay as well, by introducing subloading surface concept by Hashiguchi (1980) into the Cam-clay. In order to take into consideration the influence of density and/or confining pressure on the stress-strain behavior in 3D stress condition, the author and others also extended the above mentioned model based on $t_y$ concept, by referring to the subloading surface concept and revising it. This model is called “subloading $t_y$ model” (Nakai and Hinokio, 2004). In this paper, a new and clear explanation is given about the subloading $t_y$ model, and a simple method to consider the influence of the intermediate principal stress is introduced.

Assume the same stress change as that in Fig. 18 – i.e., from the initial state I ($t_N^{(0)}=t_N^{(0)}$, $X=0$) to the current state $P$ ($t_N^{(0)}=t_N^{(0)}$, $X=X$). The current yield surface is given by the same function as that in Eq. (32)

\[
  f = \ln \frac{t_{N1}}{t_{N0}} + \zeta(X) - \ln \frac{t_{N1}}{t_{N0}} = 0 \tag{32bis}
\]

Fig. 27 shows the $e - \ln t_{N1}$ relation. For example, when the same stress change from point I ($t_N^{(0)}=t_N^{(0)}$, $t_N^{(0)}=t_N^{(0)}$, $X=0$) to point $P$ ($t_N^{(0)}=t_N^{(0)}$) occurs for normally consolidated soil and over consolidated soil, the void ratio of normally consolidated soil changes from $e_{N0}$ to $e_0$ and the void ratio of over consolidated soil changes from $e_{N0}$ to $e_0$. The difference between normally and over consolidated soils, then, is expressed as the change from $\rho_0 (=e_{N0}-e_0)$ to $\rho (=e_0-e_0)$. Here, it can be assumed that the recoverable change of void ratio $\Delta e$ (elastic component) for normally and over consolidated soils is the same and given by the following expression using the swelling index $\kappa$:

\[
  (\Delta e)^f = \kappa \ln \frac{t_{N1}}{t_{N0}} \tag{45}
\]

so that the plastic change of void ratio $\Delta e^p$ of over consolidated soil is obtained on referring to Fig. 27.

\[
  \begin{align*}
  (\Delta e)^f &= (e_0-e) - (\Delta e)^f = (e_{N0}-e_0) - (\rho_0 - \rho) - (\Delta e)^f \\
  &= (\lambda - \kappa) \ln \frac{t_{N1}}{t_{N0}} - (\rho_0 - \rho)
  \end{align*}
  \tag{46}
\]

\[
  \ln \frac{t_{N1}}{t_{N0}} = \left(\frac{\Delta e}{\lambda - \kappa} + \frac{\rho_0 - \rho}{\lambda - \kappa}\right)
  \tag{47}
\]

and the yield function of over consolidated soil is expressed as

\[
  f = \ln \frac{t_N}{t_{N0}} + \zeta(X) - \ln \frac{t_{N1}}{t_{N0}}
  = \ln \frac{t_N}{t_{N0}} + \zeta(X) - \left(\frac{\Delta e}{\lambda - \kappa} + \frac{\rho_0 - \rho}{\lambda - \kappa}\right)
  = \ln \frac{t_N}{t_{N0}} + \zeta(X) - \frac{1}{C_p} \left(\Delta e + \rho_0 - \rho\right) = 0
  \tag{48}
\]

Equation (48) corresponds to the yield function (subloading surface) of the “subloading $t_y$ model” developed before. This yield surface expands and shrinks so that it always passes through the current stress state whenever the stress state changes in the same way as the usual subloading surface concept. Introducing the following scalars $F_\rho$ and $F_{\rho0}$ which are the functions of the current density and its initial value $\rho_0$:

\[
  F_\rho = \frac{\rho - \rho_0}{\lambda - \kappa} = \frac{\rho}{C_p} \frac{1}{1 + e_0}
  \tag{49}
\]

\[
  F_{\rho0} = \frac{\rho_0 - \rho_0}{\lambda - \kappa} = \frac{\rho_0}{C_p} \frac{1}{1 + e_0}
  \tag{50}
\]

we can rewrite Eq. (48) as

\[
  f = F(t_N, X) - \left\{ H \left(e_0^f\right) + F_{\rho0} - F_\rho \right\} = 0
  \tag{51}
\]
The consistency condition \( df=0 \) gives

\[
df = dF - (dH - dF_p)
\]

\[
df = \frac{1}{C_p} \left( \frac{d\varepsilon_p^e}{1 + \varepsilon_0} - \frac{d}{1 + \varepsilon_0} \right)
\]

\[
df_a = \frac{1}{C_p} \left( \frac{\partial f}{\partial \varepsilon_p^e} - \frac{d}{1 + \varepsilon_0} \right) = 0
\]

Equation (52)

It is then assumed that the variable \( \rho \) representing density decreases \( (d\rho<0) \) with the increase of plastic strain development and finally becomes zero (normally consolidated state). In the subloading \( t_{ij} \) model (Nakai and Hinokio, 2004), to satisfy this condition, the evolution rule of \( \rho \) is given by

\[
d\left( \frac{\rho}{1 + \varepsilon_0} \right) = \Lambda \cdot L(\rho, t_y) = \Lambda \cdot \frac{-G(\rho)}{t_y} < 0
\]

Equation (53)

Here, \( G(\rho) \) is a increasing function of \( \rho \) with satisfying \( G(0)=0 \), such as \( G(\rho) = a \rho^2 \) as shown in Fig. 28.

![Fig. 28 Function to describe degradation of \( \rho \)](image)

The loading condition of soil is presented as follows, in the same way as Hashiguchi (1980), Asaoka et al. (1997) and others:

\[
\begin{align*}
\frac{d\varepsilon_p^e}{\varepsilon_{0p}} & \neq 0 \quad \text{if} \quad \Lambda = \frac{df_a}{h} > 0 \\
\frac{d\varepsilon_p^e}{\varepsilon_{0p}} & = 0 \quad \text{if} \quad \Lambda = \frac{df_a}{h} \leq 0
\end{align*}
\]

Equation (54)

or using the symbol \(< >\) which denotes the Macaulay bracket, i.e., \(<A>=A \text{ if } A>0 ; \text{ otherwise } <A>=0\)

\[
d\varepsilon_p^e = (\Lambda) \left( \frac{\partial f}{\partial \varepsilon_p^e} \right) = \left( \frac{d\varepsilon_p^e}{h^e} \right) \frac{\partial f}{\partial t_y}
\]

Equation (55)

From Eqs. (52) and (53), the proportionality constant \( \Lambda \) is expressed as

\[
\Lambda = \frac{df}{dF} = \frac{df_a}{h} = \frac{1}{C_p} \left( \frac{\partial f}{\partial \varepsilon_p^e} + G(\rho) \right)
\]

Equation (56)

Additionally, Eq. (51) is expressed as

\[
F(t_y, X) + F_p = H(\varepsilon_p^e) + F_{0p}
\]

Equation (57) represented in term of the relation between \( F \) and \( (H+F_{0p}) \), is indicated by the dash-dotted line in Fig. 29. This line is approaching the broken line \( (F=H) \) for normally consolidated soil, with the development of plastic deformation. Equation (53) expresses the condition that \( F_p=\rho(\lambda-\kappa) \) decreases monotonously from \( F_{0p}=\rho_0(\lambda-\kappa) \) to zero with increasing plastic strain. Here, \( F_p=\rho(\lambda-\kappa) \) refers to the difference of void ratio between over consolidated soil and normally consolidated soil at the same stress condition. Furthermore, the tangential slope \( dF/dH \) of the dash-dotted line gives an idea of the stiffness against the plastic volumetric strain (or void ratio) for over consolidated soil. This can be compared with the stiffness of a normally consolidated soil (for a similar change of the yield surface), given by the slope \( dF/dH \) of the dashed line, which is always unity in this diagram.

![Fig. 29 Explanation of \( F \) and \( H \) on over consolidated soil or denser soil](image)

The elastic strain increment is usually calculated using Eqs. (17) and (18). However, it is more reasonable to calculate the elastic strain increment using \( t_{ij} \) concept as follows:

The elastic strain increment \( d\varepsilon_p^e \) is given by the following equation. The Hooke’s law is modified in such a way that the elastic volumetric strain is governed not by usual mean stress \( p \) but by mean stress \( t_{nj} \) based on \( t_{ij} \) concept in the same way as the plastic strain:

\[
d\varepsilon_p^e = \frac{1 + \nu_e}{E_e} d\left( \frac{\sigma_{ij}}{1 + X^2} \right) \delta_y
\]

Equation (58)

This is because the following equation always holds:

\[
t_{ij} = \frac{p}{1 + X^2}
\]

Equation (59)

Elastic modulus \( E_e \) is expressed in terms of the swelling index \( \kappa \) of \( e - \ln t_y \) relation and Poisson’s ratio \( \nu_e \) as

\[
E_e = \frac{3(1-2\nu_e)(1+\varepsilon_0)t_y}{\kappa}
\]

Equation (60)
4.2 Validation against test data

Tables 2 and 3 show the values of material parameters for saturated Fujinomori clay and Toyoura sand, respectively. As indicated in the tables, one parameter is added to the parameters which are fundamentally the same as those of the Cam clay model. The parameters except parameter ‘a’ can be obtained from consolidation and shear tests on normally consolidated soils. Parameter ‘β’, which represents the shape of yield surface, can be determined from the observed stress-strain-dilatancy curve or stress-dilatancy relation in shear tests, and the other parameter ‘a’ can be determined from the observed strength and/or stress-strain curves of soils with different initial void ratios.

Table-2 Material parameters for Fujinomori clay

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i = \lambda/(1+\epsilon_0)$</td>
<td>$5.08 \times 10^2$</td>
</tr>
<tr>
<td>$C_s = \kappa/(1+\epsilon_0)$</td>
<td>$1.12 \times 10^2$</td>
</tr>
<tr>
<td>$N = \epsilon_{ve}/(\sigma - q)$</td>
<td>0.83</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>3.5</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.5</td>
</tr>
<tr>
<td>$a$</td>
<td>500</td>
</tr>
</tbody>
</table>

Table-3 Material parameters for Toyoura sand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.07</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.0045</td>
</tr>
<tr>
<td>$N = \epsilon_{ve}/(\sigma - q)$</td>
<td>1.1</td>
</tr>
<tr>
<td>$R_{ls}(\sigma - q)/\sigma_{comp}$</td>
<td>3.2</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.0</td>
</tr>
<tr>
<td>$a$</td>
<td>30</td>
</tr>
</tbody>
</table>

Figure 30 shows the observed and calculated results of triaxial compression and extension tests on clay with different over consolidation ratios (OCR=1, 2, 4 and 8). Here, tests with OCR=8 are carried out under $p=98$ kPa, and the other tests are under $p=196$ kPa. The observed (symbols) and calculated (curves) results in these figures are arranged in terms of the relation between stress ratio $q/p$, deviatoric strain $\epsilon_d$ and volumetric strain $\epsilon_v$. In each figure, diagram (a) shows the results under triaxial compression condition, and diagram (b) shows those under triaxial extension condition. It can be seen from these figures that the model is capable of describing uniquely not only the influence of over consolidation ratio (density) on the deformation, dilatancy and strength of clay but also the influence of intermediate principal stress on them. Figure 31 shows the observed and calculated results of constant mean principal tests on dense and loose sands. It can be observed from these figures that the present model can describe the stress-strain-strength behavior for sands under triaxial compression and extension conditions from dense state to loose state with unified material parameters, in the same way as clay.
Figures 32 and 33 show the observed (marked by symbols) and calculated (marked by curves) variations of the three principal strains ($\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$) and the volumetric strain $\varepsilon_v$ against stress ratio $q/p$ in true triaxial tests ($\theta=15^\circ$, $30^\circ$ and $45^\circ$) on clay and dense sand under constant mean principal stress ($p=196\text{kPa}$). In each figure, $\theta$ denotes the angle between $\sigma_1$-axis and the corresponding radial stress path on the octahedral plane, as mentioned before. As can be seen from these figures, the present model predicts well stress-strain behavior not only of clay with negative dilatancy but also of sand with negative and positive dilatancy under three different principal stresses. From diagrams (a) and (b) in each figure, i.e., $\varepsilon_2$ is negative in diagram (a) but is positive in diagram (b), so it can be presumed that plane strain condition ($\varepsilon_2=0$) lies within $15^\circ<\theta<30^\circ$ for both clay and sand, which is in agreement with the results reported by many researchers. Figures 34 and 35 show the calculated directions of strain increments on the octahedral plane in the above mentioned true triaxial tests on clay and sand, corresponding to the observed results in Figs. 10 and 11. The calculated results describe well the observed direction of the strain increment on the octahedral plane, including the leftward deviation from the radial direction with the increase of stress ratio.
Figures 36 and 37 show the results and stress paths of drained cyclic triaxial tests on normally consolidated clay. As shown in stress path of diagram (c) in each figure, Fig. 36 refers to the cyclic constant mean principal stress test under constant amplitude of stress ratio, and Fig. 37 refers to the results of the cyclic constant mean principal stress test under increasing stress ratio with number of cycles. Observe that in spite of using an isotropic hardening law, the model can describe cyclic behavior of clay. This is due to the adoption of the subloading surface concept and the loading condition in Eq. (54). The reason why the present model can describe the behavior of clays as they become stiffer with increasing number of cycles is because the state variable $\rho$ increases under cyclic loadings, even if the clay is initially normally consolidated ($\rho_0=0$).

5 DESCRIPTION OF BEHAVIOR OF STRUCTURED SOILS

5.1 Extension of subloading $t_{ij}$ model to structured soils

Natural clay behaves intricately compared with remolded clay which is used in laboratory tests, because natural clay develops a complex structure in its deposition process. Such structured clay can exist in a region where its void ratio is looser than that of non-structured normally consolidated clay under the same stress condition. Such type of structured clay shows more brittle and more compressive behavior than non-structured clay. Asaoka et al. (2002) and Asaoka (2005) developed a model to describe such structured soils, introducing subloading surface and superloading surface concepts to the Cam-clay model. In their modeling, a factor related to the over consolidation ratio (corresponding to imaginary density) has been introduced to increase the stiffness, and a factor related to the soil skeleton structure has been introduced to decrease the stiffness. By controlling the evolution rules...
of these factors, it is possible to describe various features of consolidations and shear behaviors of structured soils.

In the present study, attention is focused in the real density and the bonding as the main factors that affect a structured soil, because it can be considered that the soil skeleton structure in a looser state than that of a normally consolidated soil is formed by bonding effects including interlocking between soil particles and others. It is also seen in the previous section that the subloading surface decreases with the development of plastic deformation. Furthermore, the horizontal distance between the dash-dotted line and the broken line implies the influence of the bonding and its initial value, in the development of plastic deformation. From Eq. (61), the yield function is obtained as

$$F(t_s, X) + F_\rho + F_\omega = H(e_\rho - F_\rho) + (F_\omega - F_\omega)$$

(61)

Here, $F_\rho$ and $F_\omega$ are the scalar function representing the influence of the bonding and its initial value, in the same way as $F_\rho$ and $F_\omega$ to consider the influence of the density. It can also be assumed that the value of $F_\omega$ decreases from $F_\omega$ to zero together with the development of plastic deformation. From Eq. (61), the yield function is obtained as

$$f = F(t_s, X) - \{H(e_\rho - F_\rho) + (F_\omega - F_\omega)\} = 0$$

(62)

Typical relation between $F$ and $H$ is shown in Fig. 38. The horizontal distance between the solid line and the dash-dotted line indicates the magnitude of the bonding effect $F_\omega$ starts from the initial value $F_\omega$, and decreases with the development of plastic deformation. Furthermore, the horizontal distance between the dash-dotted line and the broken line implies the influence of the current density $\rho$, which is given by Eq. (49). By using the imaginary density $\omega$ to consider the effect of the bonding, then $F_\omega$ and $F_\omega$ can be expressed with the following equations in the same way as $F_\rho$ and $F_\omega$:

$$F_\omega = \frac{\omega}{\lambda - \kappa} = \frac{1}{C_p} \frac{\omega}{1 + e_0}$$

(63)

$$F_\omega = \frac{\omega_0}{\lambda - \kappa} = \frac{1}{C_p} \frac{\omega_0}{1 + e_0}$$

(64)

Substituting Eqs. (34), (15), (49), (50), (63) and (64) into (62), we obtain

$$f = F(t_s, X) - \{H(e_\rho - F_\rho) + (F_\omega - F_\omega)\}$$

(65)

Compatibility condition ($df=0$) gives

$$df = dF - (dH - dF_\rho - dF_\omega)$$

(66)

Considering the value of $\omega$ decreases with the development of plastic deformation, the evolution rule of $\omega$ can be expressed as follows in the same way as $\rho$ in Eq. (53):

$$df = \frac{1}{C_p} \left(\frac{\rho}{1 + e_0} - d\left(\frac{\rho}{1 + e_0}\right) - d\left(\frac{\omega}{1 + e_0}\right)\right) = 0$$

(67)

$Q(\omega)$ is a monotonously increasing function of $\omega$ satisfying $Q(0)=0$, such as $Q(\omega) = b\omega$ ($b$: material parameter). As there is a possibility of $\rho$ becoming negative, the evolution rule of $\rho$ in Eq. (53) should be a function which satisfies the condition that $\rho$ converges to zero regardless of the sign of $\rho$. To satisfy this condition, the $G(\rho)$ in Fig. 28 is extended as $G(\rho) = \text{sign}(\rho) \cdot a \rho^2$. The shape of these functions $G(\rho)$ and $Q(\omega)$ used in the present simulation are shown in Fig. 39.

It should be mentioned that any kinds of increasing functions are acceptable for $G(\rho)$ and $Q(\omega)$ to describe the degradation of $\rho$ and $\omega$. Substituting Eqs. (53) and (67) into (66), the proportionality constant $\Lambda$ can be expressed as

$$\Lambda = \frac{\frac{dF}{h^2}}{\frac{1}{C_p} \left(\frac{\partial G(\rho)}{\partial \rho} + \frac{Q(\omega)}{t_N}\right)}$$

(68)
Figure 40 shows schematic illustration of variations of $F_\rho$, $F_\omega$ and $(F_\rho + F_\omega)$ for structured soil in the case that the degradation of the bonding effect is not so fast when plastic strains develop. The horizontal distance of the dotted line from the broken straight line implies the effect of the bonding $F_\omega$, which decays monotonously from $F_\omega^0$ to zero. The distance of the dash-dotted line from the broken straight line implies the effect of the density $F_\rho = \rho / (\lambda - \kappa)$. Therefore, the horizontal distance of the solid line from the broken straight line represents both effects $(F_\rho + F_\omega)$. It can be seen from this figure that the effect of the density $F_\rho$ degrades but does not converge to zero monotonically. It decreases from $F_\rho^0$ to some negative value and then converges to zero. Here, $F_\rho = \rho / (\lambda - \kappa)$ implies the difference of void ratio between structured soil and non-structured normally consolidated soil at the same stress condition, in the same way as that in Fig. 29. On the other hand, qualitative difference of the stiffness of plastic volumetric strain (or void ratio) for structured soil against the change of the yield surface is indicated not by tangential slope of the dash-dotted line but by the solid line.

The increment of real density for structured soil, therefore, is given by the following equation as the total effect of $F_\rho + F_\omega$:

$$\Delta \left( \rho \right) = d \left( \rho / (1 + e_0) \right) + d \left( \omega / (1 + e_0) \right) \quad (69)$$

Therefore, the plastic strain increment $d\varepsilon_p^p$ is obtained using the yield function in Eq. (65), the evolution rule of the density and the bonding in Eqs. (53) and (67) and assuming the flow rule in Eq. (55). The loading condition is given by Eq. (54).

5.2 Numerical simulation of structured soil by proposed model

In order to check the validity of the proposed model, numerical simulations of oedometer tests and undrained triaxial compression and extension tests for structured soil (for Fujinomori clay) are carried out. For considering the effect of the bonding, only one parameter $'b'$ is added to the parameters for the Fujinomori clay in Table 2. Here, the value of $b$ for the Fujinomori clay is 200. As mentioned before, applicability of the model for non-structured normally and over consolidated clays has been checked using various kinds of consolidation and shear tests in general stress conditions (Nakai and Hinokio, 2004). Here, two series of numerical simulations for structured soils are shown – one is under the condition with the same initial bonding $\omega_0$ and different initial void ratio $e_0$ (initial density $\rho_0$), and the other is under the condition with the same initial void ratio $e_0$ (initial density $\rho_0$) and different initial bonding $\omega_0$.

Figure 41 shows the calculated results of the oedometer tests for the clay using the same initial bonding effect ($\omega_0=0.2$) and different initial void ratio, arranged with respect to the relation of void ratio ($e$) and vertical stress in logarithmic scale (ln $\sigma_v^p$). In this figure, the open circles are the results of non-structured normally consolidated clay ($\rho_0=0$ and $\omega_0=0$). Therefore, it is possible the existence of bonding effect for the clay in the region looser than the non-structured normally consolidated clay. Furthermore, over consolidated clays with bonding show stiff response with the increase of vertical stress at the first stage, pass through the normally consolidation line (NCL) and converge to NCL from looser state than the non-structured normally consolidated clay. This is a typical consolidation behavior of aged clay and structured clay.

![Fig. 41 Calculated results of oedometer tests on clays with different initial void ratio but the same initial bonding](image)
Figure 42 shows the calculated results of undrained triaxial compression and extension tests of the same clays as those in Fig. 41. Diagram (a) shows the stress-strain relation, and diagram (b) shows the effective stress paths of these tests. The upper part of these figures is the results under triaxial compression condition, and lower part is the results under triaxial extension condition. The straight lines from the origin in diagram (b) represent the critical state lines (CSL) in $p-q$ plane. It is seen from these figures that the results can describe typical undrained shear behavior of structured soil, e.g., increasing and successive decreasing of deviatoric stress with monotonous decreasing of mean stress (when $\rho_0$ is negative), and rewinding of stress path after increasing of deviatoric and mean stresses (when $\rho_0$ is positive) in diagram (b). The difference of the behavior between triaxial compression and extension conditions can be described by using $t_{ij}$ concept.

Figures 43 and 44 show the calculated results of oedometer tests and undrained triaxial compression and extension tests for over consolidated clays which have the same initial void ratio but have different initial bonding effects, arranged in terms of the same relations as those in Figs. 41 and 42. Here, open circles show the results for clay with $\omega_0=\omega$ (non-structured over consolidated clay), which are the same as the results calculated by the subloading $t_{ij}$ model. We can see from Fig. 43 that though the density $\rho (=e_N - e)$, which is represented by the vertical distance between current void ratio and void ratio at NCL, of the clay without bonding ($\omega_0=\omega$) decreases monotonically and converges to NCL, the density of the clays with bonding ($\omega_0>\omega$) decreases to some negative values and converges to NCL from negative side of $\rho$ with a sharp reduction of the bulk stiffness. These are the typical consolidation behavior of structured clay.

Under undrained shear condition, clays with bonding are stiffer and have higher strength than clay without bonding. It is also seen from Fig. 44 that over consolidated clays without bonding show strain hardening with decrease and subsequent increase of mean stress, whereas clays with bonding show not only stress hardening with decrease and increase of mean stress but also strain softening with decrease of mean stress and deviatoric stress under undrained condition. These are also typical behavior of structured soil. From above analyses, it is noticed that the behavior of structured clays can be modeled considering the effects of density and bonding.

It can be understood that the subloading $t_{ij}$ model mentioned in the previous section is extended to one which describes “feature (vi) - behavior of structured soil” as well, by replacing the effect of the bonding by employing the imaginary increase of density and combining it with the effect of the real density.
6 DESCRIPTION OF INFLUENCE OF STRESS PATH DEPENDENCY OF PLASTIC FLOW IN MODELING

6.1 Discussion on the uniqueness of stress-dilatancy relation

According to the usual plasticity, the direction of plastic flow (direction of plastic strain increments) is independent of the direction of stress increments. This means that the stress-dilatancy relation is influenced by the stress paths. Figure 45 shows the observed stress-dilatancy relations of triaxial compression and extension tests on sand under \( p=\)constant, \( \sigma_3=\)constant, \( \sigma_1=\)constant and \( R=\sigma_1/\sigma_3=\) constant, which are arranged based on ordinary concept (diagram (a)) and \( t_{ij}\) concept (diagram (b)). It can be seen from diagram (b) that even in the stress-dilatancy relation based on \( t_{ij}\) concept, the strain increment ratio depends on the stress paths except at and after peak strength, though there is not much difference between the results of the triaxial compression and extension. Therefore, it can be said that the direction of plastic flow is influenced by the direction of stress increments except at and after peak strength.

![Observed stress-dilatancy relations of triaxial compression and extension tests](image)

Fig. 45 observed stress-dilatancy relations of triaxial compression and extension tests

6.2 Method to describe the influence of stress path dependency of plastic flow

In the previous models for clay and sand (\( t_{ij}\)-clay model and \( t_{ij}\)-sand model), such stress path dependency was considered by dividing the plastic strain increment into two components – the plastic strain increment \( d\varepsilon^{p(AF)}_{ij}\) satisfying the associated flow rule in \( t_{ij}\)-space as mentioned above and the isotropic plastic strain increment \( d\varepsilon^{p(IC)}_{ij}\) under increasing mean stress – in spite of using just one yield function and one strain hardening parameter (Nakai and Matsuoka, 1986; Nakai, 1989). The same method is employed in the subloading \( t_{ij}\) model to consider the stress path dependency on the direction of plastic flow. Figure 46 shows the yield surface \( f \) and its succeeding yield surface, where point A is the current stress state, and points B and C are the stress states on the same succeeding yield surface during strain hardening. When the stress state moves to point B in region II (\( df>0 \) and \( d\varepsilon \leq 0 \)), the plastic strain increment is only \( d\varepsilon^{p(AF)}_{ij}\). However, when it moves to point C in region III (\( df>0 \) and \( d\varepsilon>0 \)), the plastic strain increment is expressed as the summation of \( d\varepsilon^{p(AF)}_{ij}\) and \( d\varepsilon^{p(IC)}_{ij}\). From Eqs. (55) and (56),

![Calculated results of undrained triaxial tests on clays with the same initial void ratio but different bonding](image)

Fig. 44 Calculated results of undrained triaxial tests on clays with the same initial void ratio but different bonding
plastic volumetric strain under isotropic compression is given by

$$\varepsilon_v^p = \frac{1}{C_p} \left( \frac{1 + G(\rho)}{1 + G(\rho)} \right) \frac{\langle dt \rangle_{N}}{t_N}$$

Here, as mentioned before, the symbol < > denotes the Macaulay bracket, i.e., $< A > = A$ if $A \geq 0$; otherwise $< A > = 0$.

It is assumed that the plastic strain increment $\varepsilon_{ij}^{p(IC)}$ is a fraction $t_N/t_{N1}$ of the above isotropic component. Therefore,

$$d\varepsilon_{ij}^{p(IC)} = \Lambda^{(IC)} \frac{\delta_{ij}}{3} \frac{1}{C_p} \left( \frac{1 + G(\rho)}{1 + G(\rho)} \right) \frac{\langle dt \rangle_{N}}{t_N} \frac{\delta_{ij}}{3}$$

Then, the plastic component $d\varepsilon_{ij}^{p(AF)}$ which satisfies associated flow rule is given by the following equation as the remaining component:

$$d\varepsilon_{ij}^{p(AF)} = \Lambda^{(AF)} \frac{\partial f}{\partial \sigma_{ij}} = \frac{1}{C_p} \left( \frac{\partial f}{\partial \sigma_{ij}} + G(\rho) \right) \frac{\langle dt \rangle_{N}}{t_N} \frac{\partial f}{\partial \sigma_{ij}}$$

$$= \frac{df}{h^p} \frac{\langle dt \rangle_{N}}{t_N} \frac{\partial f}{\partial \sigma_{ij}}$$

Figure 47 shows the calculated relations between the stress ratio $\eta = q/p$ and the strain increment ratio $d\varepsilon/d\varepsilon$, and the calculated relation between the stress ratio $X = t_S/t_N$ and the strain increment ratio $d\varepsilon_S^p/d\varepsilon_S^p$, which corresponds to the observed values in Fig. 45. It can be seen from the comparison between Fig. 45 and Fig. 47 that the calculated results describe well the observed tendency of the stress path dependency of stress-dilatancy relation (influence of stress path on the direction of plastic flow), by dividing plastic strain increment into two components without new material parameter.

7. CONCLUSIONS

The framework of ordinary elastoplastic models for soils such as the Cam clay model is firstly discussed. Then, some important aspects of soil behavior that cannot be properly considered by Cam clay model are discussed and alternative steps to take into account these features of soil behavior are presented.

(i) Influence of intermediate principal stress on the deformation and strength of soil
(ii) Stress path dependency on the direction of plastic strain increments
(iii) Positive dilatancy during strain hardening
(iv) Behavior of soil under cyclic loading
(v) Influence of density and/or confining pressure on the deformation and strength
(vi) Behavior of structured soil

Feature (i) is considered using $t_N$ concept, by which any ordinary model can be extended to one that describe 3D soil behavior uniquely. All the material parameter can be determined only from triaxial tests. Feature (ii) is considered by dividing the plastic strain increment into two components, even though only one yield function and strain hardening parameter is used. Features (iii) to (v) are considered referring to Hashiguchi’s subloading surface.
concept and revising it. All the material parameters are independent of density and/or confining pressure. A simple interpretation for subloading surface concept when it is adapted to soil model is presented. Feature (vi) is considered by extending the above interpretation and using not only the effect of density but also the effect of bonding.

The validity of the present methods is confirmed by the results of the monotonic and cyclic loading tests not only on normally and over consolidated clay but also on loose and dense sands in three-dimensional stresses and numerical simulations.

The present model (subloading $t_q$ model) in which features (i) to (v) are taken into consideration has been applied to various kinds of geotechnical problems such as tunneling (Shahin et al., 2004a and 2004b; Sung et al., 2006) open excavation (Nakai et al., 2007; Iwata et al., 2007), soil improvement (Farias et al., 2005), bearing capacity (Nakai et al., 2005), reinforced soil (Nakai et al., 2007) and others. In every analysis, the computed results are compared with the results of the model tests and field data. Further, the following papers in this conference also describe some applications of the present model to geotechnical problems: tunneling problem (Shahin et al., 2007), retaining wall problem (Nakai et al., 2007) and bearing capacity problem (Hinokio et al., 2007).

AKNOWLEGEMENTS

The author would like to thank Prof. Feng Zhang, Dr. Masaya Hinokio, Dr. Hossain Md. Shahin and Dr. Mamoru Kikumoto of Nagoya Institute of Technology and Prof. Marcio Muniz de Farias of the University of Brasilia for their useful discussions, comments and supports regarding this paper. The author is also indebted to the previous and current students in geotechnical group of Nagoya Institute of Technology for their experimental and computational assistance and useful discussions. The author is also encouraged by the work done at Nagoya University (Prof. Asaoka and his colleague) to extend the model for structured soil.

REFERENCES


**APPENDIX**

Partial derivatives of yield function and stress variables

Generally yield function of isotropic hardening model based on $t_{ij}$-concept is given by the following form as a function of mean stress $t_N$ and stress ratio $X$

$$f = f(t_N, X) = \ln \frac{f_X}{f_{N0}} + \zeta'(X) - \ln \frac{f_{N1}}{f_{N0}} = 0 \quad (32\text{bis})$$

and definitions of tensors and scalars related to in $t_{ij}$-concept are shown in Table 1. We will firstly show the derivatives of $f$ with respect to modified stress $t_{ij}$

$$\frac{\partial f}{\partial t_{ij}} = \frac{\partial f}{\partial t_{ij}^N} + \frac{\partial f}{\partial X} \frac{\partial X}{\partial t_{ij}} \quad (A1)$$

$$\frac{\partial f}{\partial t_N} = \frac{1}{t_N} \quad (A2)$$

$$\frac{\partial t_N}{\partial t_{ij}} = \frac{\partial (t_{ij} a_{ij})}{\partial t_{ij}} = a_{ij} \quad (A3)$$

$$\frac{\partial f}{\partial X} = \zeta'(X) \quad (A4)$$

$$\frac{\partial X}{\partial t_{ij}} = \frac{\partial (\sqrt{x_{ij} x_{ij}})}{\partial x_{ij}} - \frac{x_{ij} x_{ij}}{\partial \sigma_{ij}} = 1 - X' = X - (x_{ij} x_{ij}) \quad (A5)$$

Next, the derivative of $f$ with respect to the ordinary stress $\sigma_{ij}$ is expressed as follows:

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial f}{\partial t_{ij}} \frac{\partial t_{ij}}{\partial \sigma_{ij}} + \frac{\partial f}{\partial X} \frac{\partial X}{\partial \sigma_{ij}} \quad (A6)$$

$$\frac{\partial t_{ij}}{\partial \sigma_{ij}} = \frac{3 l_i}{l_i} \quad (A7)$$

$$\frac{\partial X}{\partial \sigma_{ij}} = \frac{3 l_i}{9 l_i} \quad (A8)$$

where, $l_i$, $l_2$, and $l_3$ are the first, second and third invariants of $\sigma_{ij}$ as given by Eq. (24). Therefore, the terms $df$ and $dt_N$ can be given using general stress increment $d\sigma_{ij}$.